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An Interactive Decomposition Algorithm for Two-Level Large Scale Linear Multiobjective Optimization Problems with Stochastic Parameters Using TOPSIS Method

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ABSTRACT
This paper extended TOPSIS (Technique for Order Preference by Similarity Ideal Solution) method for solving Two-Level Large Scale Linear Multiobjective Optimization Problems with Stochastic Parameters in the right-hand side of the constraints $(TL\text{-}LSLMOP\text{-}SP)_{rhs}$ of block angular structure. In order to obtain a compromise (satisfactory) solution to the $(TL\text{-}LSLMOP\text{-}SP)_{rhs}$ of block angular structure using the proposed TOPSIS method, a modified formulas for the distance function from the positive ideal solution (PIS) and the distance function from the negative ideal solution (NIS) are proposed and modeled to include all the objective functions of the two levels. In every level, as the measure of “Closeness” d_p-metric is used, a k-dimensional objective space is reduced to two –dimentional objective space by a first-order compromise procedure. The membership functions of fuzzy set theory is used to represent the satisfaction level for both criteria. A single-objective programming problem is obtained by using the max-min operator for the second –order compromise operaion. A decomposition algorithm for generating a compromise (satisfactory) solution through TOPSIS approach is provided where the first level decision maker (FLDM) is asked to specify the relative importance of the objectives. Finally, an illustrative numerical example is given to clarify the main results developed in the paper.
***Keywords* -** Stochastic Multiobjective Optimization, Two-level decision Making Problems, TOPSIS Method, Decomposition Techniques, Fuzzy Sets

I. INTRODUCTION

In real world decision situations, when formulating a Large Scale Linear Multiobjective Optimization (LSLMO) problem, some or all of the parameters of the optimization problem are described by stochastic (or random or probabilistic) variables rather than by deterministic quantities. Most of LSLMO problems arising in applications have special structures that can be exploited. There are many familiar structures for large scale optimization problems such as: (i) the block angular structure, and (ii) angular and dual- angular structure to the constraints, and several kinds of decomposition methods for linear and nonlinear programming problems with those structures have been proposed in [2, 12, 13, 27, 29, 35, 43,44, 45, 49].

Two-Level Large Scale Linear Multiobjective Optimization (TL-LSLMO) Problems with block angular structure consists of the objectives of the leader at its first level and that is of the follower at the second level. The decision maker (DM) at each level attempts to optimize his individual objectives, which usually depend in part on the variables controlled by the decision maker (DM) at the other levels and their final decisions are executed sequentially where the FLDM makes his decision firstly. The research and applications concentrated mainly on two-level programming (see f. i. [7, 8, 11, 14, 15, 16, 18, 19, 20, 21, 22, 24, 25, 26, 31,32, 33, 48,52,53]).

TOPSIS was first developed by C. L. Hwang and K. Yoon [37] for solving a multiple attributes decision making (MADM) problems. It is based upon the principle that the chosen alternative should have the shortest distance from the PIS and the farthest from the NIS. T. H. M. Abou-El-Enien [4] presents many algorithms for solving different kinds of LSLMO problems using TOPSIS method.

M. A. Abo-Sinna and T. H. M. Abou-El-Enien [9] extended TOPSIS approach to solve large scale multiple objectives decision making (LSMODM) problems with block angular structure.

T. H. M. Abou-El-Enien [3] extended TOPSIS method for solving large scale integer linear vector optimization problems with chance constraints (CHLSILVOP) of a special type.

I. A. Baky and M. A. Abo-Sinna [20] proposed a TOPSIS algorithm for bi-level multiple objectives decision making (BL-MODM) problems.

T. H. M. Abou-El-Enien [5] presents an interactive TOPSIS algorithm for solving a special type of linear fractional vector optimization problems.

P.P. Dey *et al.* [28] extend TOPSIS for solving linear fractional bi-level multi-objective decision-making problem based on fuzzy goal programming.

Recently, M. A. Abo-Sinna and T. H. M. Abou-El-Enien [11] extend TOPSIS for solving Large Scale Bi-level Linear Vector Optimization Problems (LS-BL-LVOP), they further extended the concept of TOPSIS [Lia *et al.* (45)] for LS-BL-LVOP.

This paper extended TOPSIS method for solving $(\text{TL-LSLMOP-SP})_{rhs}$ of block angular structure. Also, the concept of TOPSIS is extended [Lia *et al.* (41)] for $(\text{TL-LSLMOP-SP})_{rhs}$ of block angular structure.

The following section will give the formulation of $(\text{TL-LSLMOP-SP})_{rhs}$ of block angular structure. The family of d_p-distance and its normalization is discussed in section 3. The TOPSIS approach for $(\text{TL-LSLMOP-SP})_{rhs}$ of block angular is presented in section 4. By use of TOPSIS, a decomposition algorithm is proposed for solving $(\text{TL-LSLMOP-SP})_{rhs}$ of block angular structure in section 5. An illustrative numerical example is given in section 6. Finally, summary and conclusions will be given in section 7.

II. Formulation of $(\text{TL-LSLMOP-SP})_{rhs}$ of block angular structure

Consider there are two levels in a hierarchy structure with a first level decision maker (FLDM) and second level decision maker (SLDM). Let the $(\text{TL-LSLMOP-SP})_{rhs}$ of the following block angular structure :

[FLDM]

$$\underset{X_{I_1}}{Maximize}\ Z_{I_1}(X_{I_1},X_{I_2}) = \underset{X_{I_1}}{Maximize}\left(z_{I_1 1}(X_{I_1},X_{I_2}),\ldots,z_{I_1 k_{I_1}}(X_{I_1},X_{I_2})\right) \quad (1\text{-a})$$

$$where\ X_{I_2}\ solves\ second\ level$$

[SLDM]

$$\underset{X_{I_2}}{Maximize}\ Z_{I_2}(X_{I_1},X_{I_2}) = \underset{X_{I_2}}{Maximize}\left(z_{I_2 1}(X_{I_1},X_{I_2}),\ldots,z_{I_2 k_{I_2}}(X_{I_1},X_{I_2})\right) \quad (1\text{-b})$$

subject to

$$X \in M = \{P\{\textstyle\sum_{j=1}^{q}\sum_{i=1}^{n} a_{ij\,h_0}x_{ij\,h_0} \le v_{h_0}\} \ge \alpha_{h_0}, \qquad h_0 = 1,2,3,\ldots,m_0, \quad (1\text{-c})$$

$$P\left\{\textstyle\sum_{i=1}^{n} b_{ij\,h_j}x_{ij\,h_j} \le v_{h_j}\right\} \ge \alpha_{h_j}, \qquad h_j = m_{j-1}+1, m_{j-1}+2,\ldots,m_j, \quad (1\text{-d})$$

$$x_{ij} \ge 0, i \in N, j = 1,2,\ldots,q, q > 1\}. \quad (1\text{-e})$$

where

a and b are constants,

k : the number of objective functions,

k_{I_1} : the number of objective functions of the FLDM,

k_{I_2} : the number of objective functions of the SLDM,

n_{I_1} : the number of variables of the FLDM,

n_{I_2} : the number of variables of the SLDM,

q : the number of subproblems,

m : the number of constraints,

n : the numer of variables,

n_j : the number of variables of the j^{th} subproblem, $j=1,2,\ldots,q,$

m_o : the number of the common constraints represented by $\sum_{j=1}^{q}\sum_{i=1}^{n} a_{ij\,h_0}x_{ij\,h_0} \le v_{h_0}$

m_j : the number of independent constraints of the j^{th} subproblem represented by

$$\sum_{i=1}^{n} b_{ij\,h_j}x_{ij\,h_j} \le v_{h_j}, j = 1,2,\ldots,q, q > 1,$$

A_j : an $(m_o \times n_j)$ coefficient matrix,

D_j : an $(m_j \times n_j)$ coefficient matrix,

b_o : an m_o-dimensional column vector of right-hand sides of the common constraints whose elements are constants,

b_j : an m_j-dimensional column vector of independent constraints right-hand sides whose elements are the constants of the constraints for the j^{th} subproblem, $j=1,2,\ldots,q,$

C_{ij} : an n_j-dimensional row vector for the j^{th} subproblem in the i^{th} objective function,

R : the set of all real numbers,

X : an n-dimensional column vector of variables,

X_j : an n_j -dimensional column vector of variables for the j^{th} subproblem, $j=1,2,\ldots,q,$

X_{I_1} : an n_{I_1} - dimensional column vector of variables of the FLDM,

X_{I_2} : an n_{I_2} - dimensional column vector of variables of the SLDM,

$K = \{1,2,\ldots,k\}$

$N = \{1,2,\ldots..,n\}$,
$R^n = \{X=(x_1, x_2,\ldots,x_n\}^T : x_i \in R, i \in N\}$.

If the objective functions are linear, then the objective function can be written as follows:

$z_i(X)=\sum_{j=1}^{q} z_{ij} =\sum_{j=1}^{q} C_{ij} X_j$, $i=1,2,\ldots,k$ (2)

In addition, P means probability, α_{h_0} and α_{h_j} are a specified probability levels. For the sake of simplicity, consider that the random parameters, v_{h_0} and v_{h_j} are distributed normally and independently of each other with known means E$\{v_{h_0}\}$ and E$\{v_{h_j}\}$ and variances Var$\{v_{h_0}\}$ and Var$\{v_{h_j}\}$.

Using the chance constrained programming technique [3], the deterministic version of problem (1) can be written as follows :

[FLDM]

$$\underset{X_{I_1}}{Maximize}\ Z_{I_1}(X_{I_1}, X_{I_2}) = \underset{X_{I_1}}{Maximize}\left(z_{I_1 1}(X_{I_1}, X_{I_2}), \ldots., z_{I_1 k_{I_1}}(X_{I_1}, X_{I_2})\right) \quad (3\text{-a})$$

$where\ X_{I_2}\ solves\ second\ level$

[SLDM]

$$\underset{X_{I_2}}{Maximize}\ Z_{I_2}(X_{I_1}, X_{I_2}) = \underset{X_{I_2}}{Maximize}\left(z_{I_2 1}(X_{I_1}, X_{I_2}), \ldots., z_{I_2 k_{I_2}}(X_{I_1}, X_{I_2})\right) \quad (3\text{-b})$$

subject to

$$X \in M^{/} = \{\sum_{j=1}^{q}\sum_{i=1}^{n} a_{ij\,h_0} x_{ij\,h_0} \leq E\{v_{h_0}\} + k_{\alpha_0}\sqrt{Var\{v_{h_0}\}},\ h_0 = 1,2,\ldots.,m_0, \quad (3\text{-c})$$

$$\sum_{i=1}^{n} b_{ij\,h_j} x_{ij\,h_j} \leq E\{v_{h_j}\} + k_{\alpha_j}\sqrt{Var\{v_{h_j}\}},\ h_j = m_{j-1}+1, m_{j-1}+2, \ldots., m_j, \quad (3\text{-d})$$

$$x_{ij} \geq 0, i \in N, j = 1,2,\ldots,q, q > 1\}. \quad (3\text{-e})$$

where k_{α_j}, j=0,1,2,….,q, is the standard normal value such that $\Phi(k_{\alpha_j})$=1- α_j, j=0,1,…,q, and Φ represents the cumulative distribution function of the standard normal distribution.

III. Some Basic Concepts of distance Measures

The compromise (satisfactory) programming approach [17, 34, 36, 39, 40, 41, 42, 53, 54, 56] has been developed to perform multiobjective optimization problems, reducing the set of nondominated solutions. The compromise solutions are those which are the closest by some distance measure to the ideal one.

The point $z_i(X^*) = \sum_{j=1}^{q} z_{ij}(X^*)$ in the criteria space is called the ideal point (reference point). As the measure of "closeness", d_p-metric is used. The d_p-metric defines the distance between two points, $z_i(X) = \sum_{j=1}^{q} z_{ij}(X)$ and $z_i(X^*) = \sum_{j=1}^{q} z_{ij}(X^*)$ (the reference point) in k-dimensional space [50] as:

$$d_p = \left(\sum_{i=1}^{k} w_i^p (z_i^* - z_i)^p\right)^{1/p} = \left(\sum_{i=1}^{k} w_i^p \left(\sum_{j=1}^{q} z_{ij}^* - \sum_{j=1}^{q} z_{ij}\right)^p\right)^{1/p} \quad (4)$$

where $p \geq 1$.

Unfortunately, because of the incommensurability among objectives, it is impossible to directly use the above distance family. To remove the effects of the incommensurability, we need to normalize the distance family of equation (4) by using the reference point [4, 36, 41] as :

$$d_p = \left(\sum_{i=1}^{k} w_i^p \left(\frac{\sum_{j=1}^{q} z_{ij}^* - \sum_{j=1}^{q} z_{ij}}{\sum_{j=1}^{q} z_{ij}^*}\right)^p\right)^{1/p} \quad (5)$$

where $p \geq 1$.

To obtain a compromise (satisfactory) solution for problem (3) , the global criteria method [40] for problem (3) [4, 45] uses the distance family of equation (5) by the ideal solution being the reference point. The problem becomes how to solve the following auxiliary problem :

$$\underset{X \in M^{/}}{Minimize}\ d_p = \left(\sum_{i=1}^{k} w_i^p \left(\frac{\sum_{j=1}^{q} z_{ij}(X^*) - \sum_{j=1}^{q} z_{ij}(X)}{\sum_{j=1}^{q} z_{ij}(X^*)}\right)^p\right)^{1/p} \quad (6)$$

where X^* is the PIS and $p = 1,2,\ldots..,\infty$.

Usually, the solutions based on PIS are different from the solutions based on NIS. Thus, both PIS(z^*) and NIS(z^-) can be used to normalize the distance family and obtain [4]:

$$d_p = \left(\sum_{i=1}^{k} w_i^p \left(\frac{\sum_{j=1}^{q} z_{ij}^* - \sum_{j=1}^{q} z_{ij}}{\sum_{j=1}^{q} z_{ij}^* - \sum_{j=1}^{q} z_{ij}^-} \right)^p \right)^{1/p} \quad (7)$$

where $p \geq 1$.

IV. TOPSIS for (TL-LSLMOP-SP)$_{rhs}$ of block angular structure

Problem (3) can be rewritten as follows [4]:

[FLDM]

$$\underset{X_{I_1}}{Maximize/Minimize}\; Z_{I_1}(X_{I_1}, X_{I_2}) = \underset{X_{I_1}}{Maximize/Minimize} \left(z_{I_1 1}(X_{I_1}, X_{I_2}), \ldots, z_{I_1 k_{I_1}}(X_{I_1}, X_{I_2}) \right)$$

$$where\ X_{I_2}\ solves\ second\ level$$

[SLDM]

$$\underset{X_{I_2}}{Maximize/Minimize}\; Z_{I_2}(X_{I_1}, X_{I_2}) = \underset{X_{I_2}}{Maximize/Minimize} \left(z_{I_2 1}(X_{I_1}, X_{I_2}), \ldots, z_{I_2 k_{I_2}}(X_{I_1}, X_{I_2}) \right)$$

subject to (8)

$X \in M^/$

where

$\sum_{j=1}^{q} z_{tj}\ (X)$: Objective Function for Maximization, $t \in K_1 \sqsubset K$,

$\sum_{j=1}^{q} z_{vj}\ (X)$: Objective Function for Minimization, $v \in K_2 \sqsubset K$.

4-1. Phase (I)

Consider the FLDM problem of problem (8):

[FLDM]

$$\underset{X_{I_1}}{Maximize/Minimize}\; Z_{I_1}(X_{I_1}, X_{I_2}) = \underset{X_{I_1}}{Maximize/Minimize} \left(z_{I_1 1}(X_{I_1}, X_{I_2}), \ldots, z_{I_1 k_{I_1}}(X_{I_1}, X_{I_2}) \right)$$

subject to (9)

$X \in M^/$

where

$\sum_{j=1}^{q} z_{tj}\ (X)$: Objective Function for Maximization, $t \in K_1 \sqsubset K$,

$\sum_{j=1}^{q} z_{vj}\ (X)$: Objective Function for Minimization, $v \in K_2 \sqsubset K$.

In order to use the distance family of equation (7) to resolve problem (9), we must first find PIS(z^*) and NIS(z^-) which are [4, 40]:

$$z^{*FLDM} = \underset{X \in M^/}{Maximize(or\ Minimize)} \sum_{j=1}^{q} z_{tj}^{FLDM}\ (X) \left(or\ \sum_{j=1}^{q} z_{vj}^{FLDM}\ (X) \right), \forall t (and\ v) \quad (10\text{-a})$$

$$z^{-FLDM} = \underset{X \in M^/}{Minimize(or\ Maximize)} \sum_{j=1}^{q} z_{tj}^{FLDM}\ (X) \left(or\ \sum_{j=1}^{q} z_{vj}^{FLDM}\ (X) \right), \forall t (and\ v) \quad (10\text{-b})$$

where $K = K_1 \cup K_2$,

$z^{*\mathrm{FLDM}} = \left(z_1^{*FLDM}, z_2^{*FLDM}, \ldots, z_{k_{I_1}}^{*FLDM} \right)$ and $z^{-\mathrm{FLDM}} = (z_1^{-FLDM}, z_2^{-FLDM}, \ldots\ldots, z_{k_{I_1}}^{-FLDM})$ are the individual positive (negative) ideal solutions for the FLDM.

Using the PIS and the NIS for the FLDM, we obtain the following distance functions from them, respectively:

$$d_P^{PIS^{\mathrm{FLDM}}} = \left(\sum_{t \in K_1} w_t^p \left(\frac{\sum_{j=1}^{q} z_{tj}^{*\mathrm{FLDM}} - \sum_{j=1}^{q} z_{tj}^{\mathrm{FLDM}}\ (X)}{\sum_{j=1}^{q} z_{tj}^{*\mathrm{FLDM}} - \sum_{j=1}^{q} z_{tj}^{-FLDM}} \right)^p + \sum_{v \in K_2} w_v^p \left(\frac{\sum_{j=1}^{q} z_{vj}^{FLDM}\ (X) - \sum_{j=1}^{q} z_{vj}^{*FLDM}}{\sum_{j=1}^{q} z_{vj}^{-FLDM} - \sum_{j=1}^{q} z_{vj}^{*FLDM}} \right)^p \right)^{1/p} \quad (11\text{-a})$$

(11-a)

and

$$d_P^{NIS^{\mathrm{FLDM}}} = \left(\sum_{t \in K_1} w_t^p \left(\frac{\sum_{j=1}^{q} z_{tj}^{FLDM}\ (X) - \sum_{j=1}^{q} z_{tj}^{-FLDM}}{\sum_{j=1}^{q} z_{tj}^{*FLDM} - \sum_{j=1}^{q} z_{tj}^{-FLDM}} \right)^p + \sum_{v \in K_2} w_v^p \left(\frac{\sum_{j=1}^{q} z_{vj}^{-FLDM} - \sum_{j=1}^{q} z_{vj}^{FLDM}\ (X)}{\sum_{j=1}^{q} z_{vj}^{-FLDM} - \sum_{j=1}^{q} z_{vj}^{*FLDM}} \right)^p \right)^{1/p} \quad (11\text{-b})$$

where $w_i = 1,2, \ldots, k$, are the relative importance (weighs) of objectives, and $p = 1,2, \ldots\ldots, \infty$.

In order to obtain a compromise solution for the FLDM, we transfer the FLDM of problem (9) into the following two-objective problem with two commensurable (but often conflicting) objectives [4, 41]:

$Minimize\ d_p^{PIS^{FLDM}}(X)$
$Maximize\ d_p^{NIS^{FLDM}}(X)$
subject to (12)
$X \in M^{/}$
where $p = 1,2, \ldots\ldots, \infty$.

Since these two objectives are usually conflicting to each other, we can simultaneously obtain their individual optima. Thus, we can use membership functions to represent these individual optima. Assume that the membership functions ($\mu_1(X)$ and $\mu_2(X)$) of two objective functions are linear. Then, based on the preference concept, we assign a larger degree to the one with shorter distance from the PIS for $\mu_1(X)$ and assign a larger degree to the one with farther distance from NIS for $\mu_2(X)$. Therefore, as shown in figure (1), $\mu_1(X) \equiv \mu_{d_p^{PIS^{FLDM}}}(X)$ and $\mu_2(X) \equiv \mu_{d_p^{NIS^{FLDM}}}(X)$ can be obtained as the following (see [1, 6, 10, 11, 23, 38, 39, 47, 49, 56]):

$$\mu_1(X) = \begin{cases} 1, & if\ d_p^{PIS^{FLDM}}(X) < \left(d_p^{PIS^{FLDM}}\right)^*, \\ 1 - \dfrac{d_p^{PIS^{FLDM}}(X) - \left(d_p^{PIS^{FLDM}}\right)^*}{\left(d_p^{PIS^{FLDM}}\right)^- - \left(d_p^{PIS^{FLDM}}\right)^*}, & if\ \left(d_p^{PIS^{FLDM}}\right)^- \geq d_p^{PIS^{FLDM}}(X) \geq \left(d_p^{PIS^{FLDM}}\right)^*, \\ 0, & if\ d_p^{PIS^{FLDM}}(X) > \left(d_p^{PIS^{FLDM}}\right)^-, \end{cases} \quad (13\text{-a})$$

$$\mu_2(X) = \begin{cases} 1, & if\ d_p^{NIS^{FLDM}}(X) > \left(d_p^{NIS^{FLDM}}\right)^*, \\ 1 - \dfrac{\left(d_p^{NIS^{FLDM}}\right)^* - d_p^{NIS^{FLDM}}(X)}{\left(d_p^{NIS^{FLDM}}\right)^* - \left(d_p^{NIS^{FLDM}}\right)^-}, & if\ \left(d_p^{NIS^{FLDM}}\right)^- \leq d_p^{NIS^{FLDM}}(X) \leq \left(d_p^{NIS^{FLDM}}\right)^*, \\ 0, & if\ d_p^{NIS^{FLDM}}(X) < \left(d_p^{NIS^{FLDM}}\right)^-, \end{cases} \quad (13\text{-b})$$

where

$$\left(d_p^{PIS^{FLDM}}\right)^* = \underset{X \in M^{/}}{Minimize}\ d_P^{PIS^{FLDM}}(X)\ \ and\ the\ solution\ is\ X^{PIS^{FLDM}},$$

$$\left(d_p^{NIS^{FLDM}}\right)^* = \underset{X \in M^{/}}{Maximize}\ d_P^{NIS^{FLDM}}(X)\ \ and\ the\ solution\ is\ X^{NIS^{FLDM}},$$

$$\left(d_p^{PIS^{FLDM}}\right)^- = d_p^{PIS^{\mathrm{FLDM}}}\left(X^{NIS^{FLDM}}\right) and \left(d_p^{NIS^{FLDM}}\right)^- = d_p^{NIS^{FLDM}}\left(X^{PIS^{FLDM}}\right).$$

Now, by applying the max-min decision model which is proposed by R. E. Bellman and L. A. Zadeh [23] and extended by H. –J. Zimmermann [56], we can resolve problem (12). The satisfying decision of the FLDM problem (9), $X^{*^{\mathrm{FLDM}}} = (X_{I_1}^{*^{\mathrm{FLDM}}}, X_{I_2}^{*^{\mathrm{FLDM}}})$, may be obtained by solving the following model:

$$\mu_D\left(X^{*^{\mathrm{FLDM}}}\right) = \underset{X \in M^{/}}{Maximize}\ \left\{Min.\left(\mu_1(X),\ \mu_2(X)\right)\right\} \quad (14)$$

Finally, if $\delta^{FLDM} = Minimize\ (\mu_1(X),\ \mu_2(X))$, the model (14) is equivalent to the form of Tchebycheff model (see [30]), which is equivalent to the following model:

$Maximize\ \gamma^{FLDM}$, (15-a)

subject to
$\mu_1(X) \geq \gamma^{FLDM}$, (15-b)
$\mu_2(X) \geq \gamma^{FLDM}$, (15-c)
$X \in M^{/}, \quad \gamma^{FLDM} \in [0,1]$, (15-d)

where δ^{FLDM} is the satisfactory level for both criteria of the shortest distance from the *PIS* and the farthest distance from the *NIS*. It is well known that if the optimal solution of (15) is the vector $\left(\gamma^{*^{FLDM}}, X^{*^{\text{FLDM}}}\right)$, then $X^{*^{\text{FLDM}}}$ is a nondominated solution [36, 39, 51, 55] of (12) and a satisfactory solution [4, 54] of the FLDM problem (9).

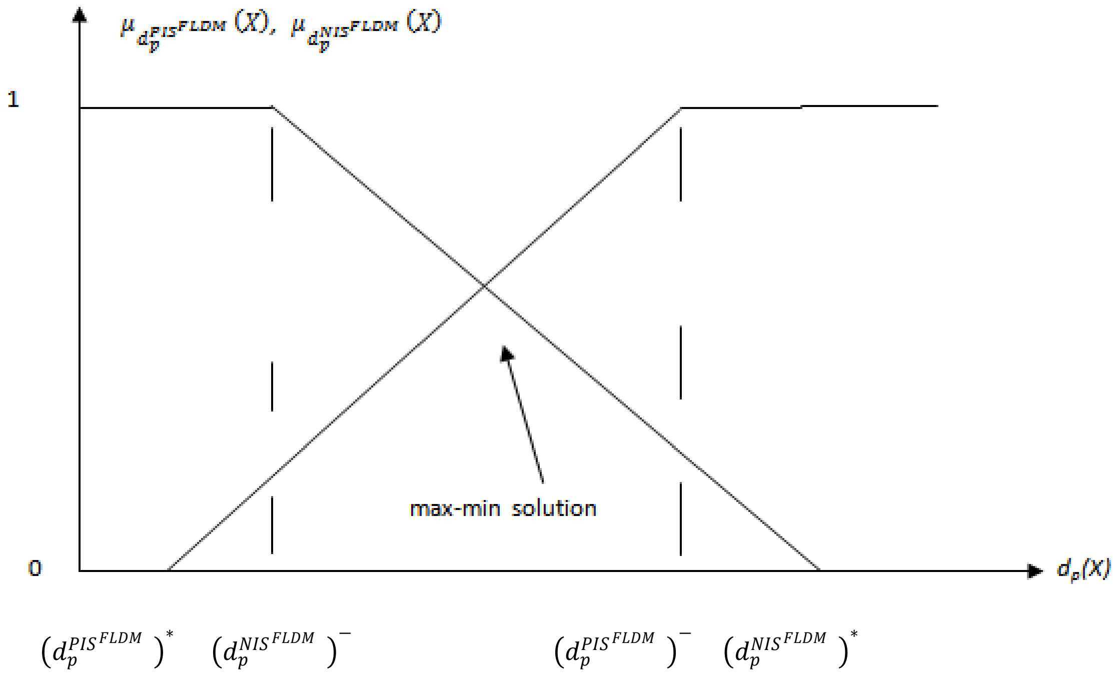

Figure (1): The membership functions of $\mu_{d_p^{PIS^{FLDM}}}(X)$ *and* $\mu_{d_p^{NIS^{FLDM}}}(X)$

The basic concept of the two-level programming technique is that the FLDM sets his/her goals and/or decisions with possible tolerances which are described by membership functions of fuzzy set theory. According to this concept, let τ_i^L and $\tau_i^R, i = 1,2,\dots,n_{I_1}$ be the maximum acceptable negative and positive tolerance (relaxation) values on the decision vector considered by the FLDM, $X_{I_1}^{*^{FLDM}} = \left(x_{I_11}^{*^{FLDM}}, x_{I_12}^{*^{FLDM}}, \dots, x_{I_1n_{I_1}}^{*^{FLDM}}\right)$. The tolerances give the SLDM an extent feasible region to search for the satisfactory solution. If the feasible region is empty, the negative and positive tolerances must be increased to give the SLDM an extent feasible region to search for the satisfactory solution, [4, 36, 54]. The linear membership functions (Figure 2) for each of the n_{I_1}components of the decision vector $\left(x_{I_11}^{*^{FLDM}}, x_{I_12}^{*^{FLDM}}, \dots, x_{I_1n_{I_1}}^{*^{FLDM}}\right)$ controlled by the FLDM can be formulated as:

$$\mu_{I_1i}(x_{I_1i}) = \begin{cases} \dfrac{x_{I_1i} - \left(X_{I_1i}^{*^{FLDM}} - \tau_i^L\right)}{\tau_i^L} & if\ x_{I_1i}^{*^{FLDM}} - \tau_i^L \le x_{I_1i} \le x_{I_1i}^{*^{FLDM}} \\ \dfrac{\left(X_{I_1i}^{*^{FLDM}} + \tau_i^R\right) - x_{I_1i}}{\tau_i^R} & if\ x_{I_1i}^{*^{FLDM}} \le x_{I_1i} \le x_{I_1i}^{*^{FLDM}} + \tau_i^R,\ i = 1,2,\dots,n_{I_1}, \\ 0 & if\ otherwise, \end{cases} \quad (16)$$

It may be noted that, the decision maker may desire to shift the range of x_{I_1i}. Following Pramanik & Roy [47] and Sinha [50], this shift can be achieved.

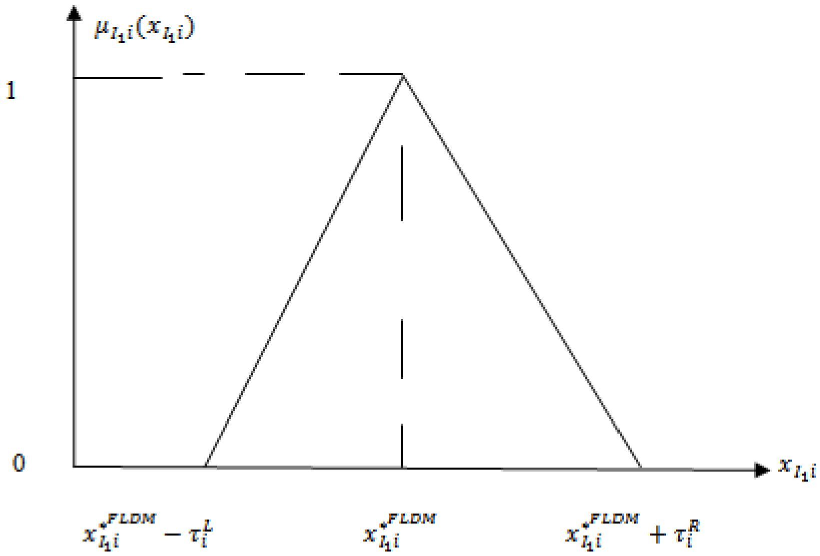

Figure (2): The membership function of the decision variable $x_{I_1 i}$

4-2. Phase (II)
The SLDM problem of problem (8) can be written as follows:
[SLDM]

$$\underset{X_{I_2}}{Maximize}\; Z_{I_2}(X_{I_1}, X_{I_2}) = \underset{X_{I_2}}{Maximize}\left(z_{I_2 1}(X_{I_1}, X_{I_2}), \ldots, z_{I_2 k_{I_2}}(X_{I_1}, X_{I_2})\right)$$

subject to (17)
$X \in M^{/}$

where
$\sum_{j=1}^{q} z_{tj}\ (X)$: Objective Function for Maximization, $t \in K_1 \subset K$,
$\sum_{j=1}^{q} z_{vj}\ (X)$: Objective Function for Minimization, $v \in K_2 \subset K$.

In order to use the distance family of equation (7) to resolve problem (17), we must first find PIS(z^*) and NIS(z^-) which are [4, 41]:

$$z^{*SLDM} = \underset{X \in M^{/}}{Maximize(or\ Minimize)} \sum_{j=1}^{q} z_{tj}\ (X)\left(or\ \sum_{j=1}^{q} z_{vj}\ (X)\right), \forall t(and\ v) \tag{18-a}$$

$$z^{-SLDM} = \underset{X \in M^{/}}{Minimize(or\ Maximize)} \sum_{j=1}^{q} z_{tj}\ (X)\left(or\ \sum_{j=1}^{q} z_{vj}\ (X)\right), \forall t(and\ v) \tag{18-b}$$

where $K = K_1 \cup K_2$,
$z^{*SLDM} = \left(z_1^{*SLDM}, z_2^{*SLDM}, \ldots, z_{k_{I_2}}^{*SLDM}\right)$ and $z^{-SLDM} = (z_1^{-SLDM}, z_2^{-SLDM}, \ldots\ldots, z_{k_{I_2}}^{-SLDM})$ are the individual positive (negative) ideal solutions for the SLDM.

In order to obtain a compromise (satisfactory) solution to problem (8) using TOPSIS approach, the distance family of (7) to represent the distance function from the positive ideal solution, $d_P^{PIS^{TL}}$, and the distance function from the negative ideal solution, $d_P^{NIS^{TL}}$, can be proposed, in this paper, for the objectives of the FLDM and the SLDM as follows:

$$d_P^{PIS^{TL}} = \left(\sum_{t \in K_1} w_t^p \left(\frac{\sum_{j=1}^{q} z_{tj}^{*FLDM} - \sum_{j=1}^{q} z_{tj}^{FLDM}(X)}{\sum_{j=1}^{q} z_{tj}^{*FLDM} - \sum_{j=1}^{q} z_{tj}^{-FLDM}}\right)^p + \sum_{v \in K_2} w_v^p \left(\frac{\sum_{j=1}^{q} z_{vj}^{FLDM}(X) - \sum_{j=1}^{q} z_{vj}^{*FLDM}}{\sum_{j=1}^{q} z_{vj}^{-FLDM} - \sum_{j=1}^{q} z_{vj}^{*FLDM}}\right)^p + \sum_{t \in K_1} w_t^p \left(\frac{\sum_{j=1}^{q} z_{tj}^{*SLDM} - \sum_{j=1}^{q} z_{tj}^{SLDM}(X)}{\sum_{j=1}^{q} z_{tj}^{*SLDM} - \sum_{j=1}^{q} z_{tj}^{-SLDM}}\right)^p + \sum_{v \in K_2} w_v^p \left(\frac{\sum_{j=1}^{q} z_{vj}^{SLDM}(X) - \sum_{j=1}^{q} z_{vj}^{*SLDM}}{\sum_{j=1}^{q} z_{vj}^{-SLDM} - \sum_{j=1}^{q} z_{vj}^{*SLDM}}\right)^p\right)^{1/p} \tag{19-a}$$

and

$$d_P^{NIS^{\text{TL}}} = \left(\sum_{t\in K_1} w_t^p \left(\frac{\sum_{j=1}^q z_{tj}^{FLDM}(X) - \sum_{j=1}^q z_{tj}^{-FLDM}}{\sum_{j=1}^q z_{tj}^{*FLDM} - \sum_{j=1}^q z_{tj}^{-FLDM}}\right)^p + \sum_{v\in K_2} w_v^p \left(\frac{\sum_{j=1}^q z_{vj}^{-FLDM} - \sum_{j=1}^q z_{vj}^{FLDM}(X)}{\sum_{j=1}^q z_{vj}^{-FLDM} - \sum_{j=1}^q z_{vj}^{*FLDM}}\right)^p + \sum_{t\in K_1} w_t^p \left(\frac{\sum_{j=1z}^q z_{tj}^{SLDM}(X) - \sum_{j=1}^q z_{tj}^{-SLDM}}{\sum_{j=1}^q z_{tj}^{*SLDM} - \sum_{j=1}^q z_{tj}^{-SLDM}}\right)^p + \sum_{v\in K_2} w_v^p \left(\frac{\sum_{j=1}^q z_{vj}^{-SLDM} - \sum_{j=1}^q z_{vj}^{SLDM}(X)}{\sum_{j=1}^q z_{vj}^{-SLDM} - \sum_{j=1}^q z_{vj}^{*SLDM}}\right)^p\right)^{1/p} \tag{19-b}$$

where $w_i = 1,2,\ldots.,k$, are the relative importance (weighs) of objectives, and $p = 1,2,\ldots..,\infty$.

In order to obtain a compromise (satisfactory) solution, we transfer problem (8) into the following two-objective problem with two commensurable (but often conflicting) objectives [4, 41]:

$Minimize\ d_p^{PIS^{\text{TL}}}(X)$
$Maximize\ d_p^{NIS^{\text{TL}}}(X)$
subject to (20)
$X \in M^/$
where $p = 1,2,\ldots..,\infty$.

Since these two objectives are usually conflicting to each other, we can simultaneously obtain their individual optima. Thus, we can use membership functions to represent these individual optima. Assume that the membership functions ($\mu_3(X)$ and $\mu_4(X)$) of two objective functions are linear. Then, based on the preference concept, we assign a larger degree to the one with shorter distance from the PIS for $\mu_3(X)$ and assign a larger degree to the one with farther distance from NIS for $\mu_4(X)$. Therefore, as shown in figure (3), $\mu_3(X) \equiv \mu_{d_p^{PIS^{\text{TL}}}}(X)$ and $\mu_4(X) \equiv \mu_{d_p^{NIS^{\text{TL}}}}(X)$ can be obtained as the following (see [1, 6, 10, 11, 23, 38, 39, 47, 49, 56]):

$$\mu_3(X) = \begin{cases} 1, & if\ d_p^{PIS^{\text{TL}}}(X) < \left(d_p^{PIS^{\text{TL}}}\right)^*, \\ 1 - \frac{d_p^{PIS^{\text{TL}}}(X) - \left(d_p^{PIS^{\text{TL}}}\right)^*}{\left(d_p^{PIS^{\text{TL}}}\right)^/ - \left(d_p^{PIS^{\text{TL}}}\right)^*}, & if\ \left(d_p^{PIS^{\text{TL}}}\right)^- \geq d_p^{PIS^{\text{TL}}}(X) \geq \left(d_p^{PIS^{\text{TL}}}\right)^*, \\ 0, & if\ d_p^{PIS^{\text{TL}}}(X) > \left(d_p^{PIS^{\text{TL}}}\right)^-, \end{cases} \tag{21-a}$$

$$\mu_4(X) = \begin{cases} 1, & if d_p^{NIS^{\text{TL}}}(X) > \left(d_p^{NIS^{\text{TL}}}\right)^*, \\ 1 - \frac{\left(d_p^{NIS^{\text{TL}}}\right)^* - d_p^{NIS^{\text{TL}}}(X)}{\left(d_p^{NIS^{\text{TL}}}\right)^* - \left(d_p^{NIS^{\text{TL}}}\right)^-}, & if \left(d_p^{NIS^{\text{TL}}}\right)^- \leq d_p^{NIS^{\text{TL}}}(X) \leq \left(d_p^{NIS^{\text{TL}}}\right)^*, \\ 0, & if d_p^{NIS^{\text{TL}}}(X) < \left(d_p^{NIS^{\text{TL}}}\right)^-, \end{cases} \tag{21-b}$$

where

$$\left(d_p^{PIS^{\text{TL}}}\right)^* = \underset{X\in M^/}{Minimize}\ d_P^{PIS^{\text{TL}}}(X)\ \ and\ the\ solution\ is\ X^{PIS^{\text{TL}}},$$

$$\left(d_p^{NIS^{\text{TL}}}\right)^* = \underset{X\in M^/}{Maximize}\ d_P^{NIS^{\text{TL}}}(X)\ \ and\ the\ solution\ is\ X^{NIS^{\text{TL}}},$$

$$\left(d_p^{PIS^{\text{TL}}}\right)^- = d_p^{PIS^{\text{TL}}}\left(X^{NIS^{\text{TL}}}\right) and \left(d_p^{NIS^{\text{TL}}}\right)^- = d_p^{NIS^{\text{TL}}}\left(X^{PIS^{\text{TL}}}\right).$$

Now, by applying the max-min decision model which is proposed by R. E. Bellman and L. A. Zadeh [23] and extended by H. –J. Zimmermann [56], we can resolve problem (20). The satisfactory solution of problem (8), $X^{*^{\text{TL}}}$, may be obtained by solving the following model:

$$\mu_D\left(X^{*^{\text{TL}}}\right) = \underset{X\in M^/}{Maximize}\ \left\{Min.\left(\mu_3(X),\ \mu_4(X)\right)\right\} \tag{22}$$

Finally, if $\delta^{\text{TL}} = Minimize\ (\mu_3(X),\ \mu_4(X))$, the model (22) is equivalent to the form of Tchebycheff model (see [30]), which is equivalent to the following model:

$Maximize\ \gamma^{\text{TL}}$, (23-a)

subject to

$\mu_3(X) \geq \gamma^{\text{TL}}$, (23-b)

$\mu_4(X) \geq \gamma^{\text{TL}}$, (23-c)

$\frac{x_{I_1 i} - \left(x_{I_1 i}^{*FLDM} - \tau_i^L\right)}{\tau_i^L} \geq \gamma^{\text{TL}}, i = 1,2,\ldots,n_{I_1}$ (23-d)

$\frac{\left(x_{I_1 i}^{*FLDM} + \tau_i^R\right) - x_{I_1 i}}{\tau_i^R} \geq \gamma^{\text{TL}}, i = 1,2,\ldots,n_{I_1}$ (23-e)

$X \in M^{/}, \quad \gamma^{\text{TL}} \in [0,1]$, (23-f)

where δ^{TL} is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS. It is well known that if the optimal solution of (23) is the vector $\left(\gamma^{*\text{TL}}, X^{*\text{TL}}\right)$, then $X^{*\text{TL}}$ is a nondominated solution of (20) and a satisfactory solution for the problem (8) [4, 11, 45].

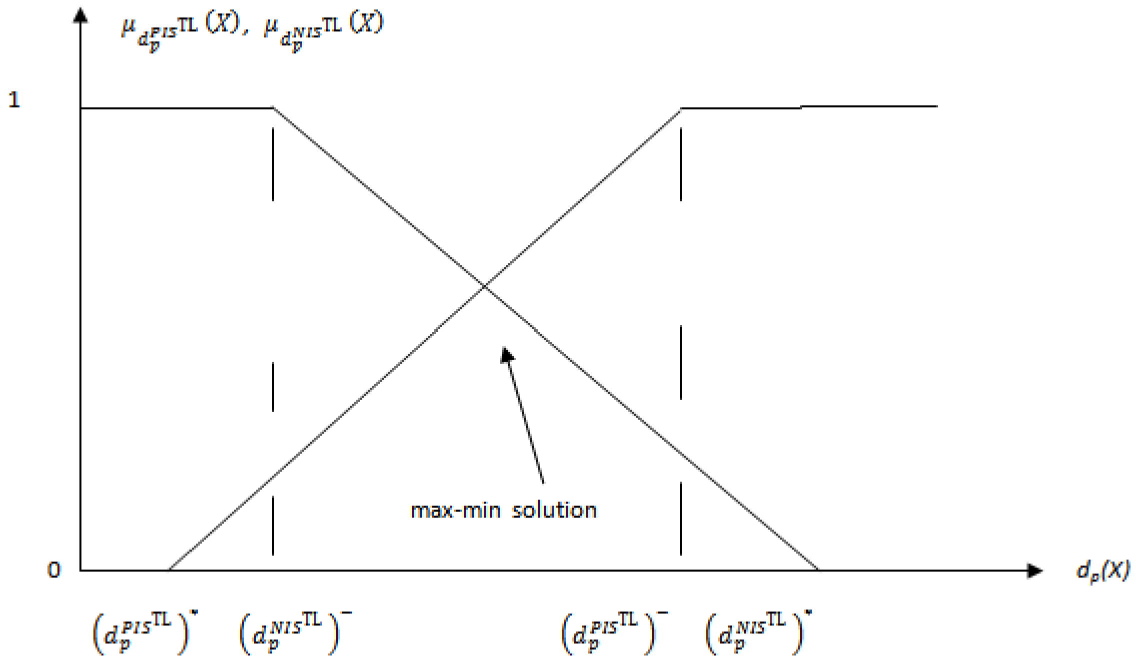

Figure (3): The membership functions of $\mu_{d_p^{PIS}\text{TL}}(X)$ *and* $\mu_{d_p^{NIS}\text{TL}}(X)$

V. A decomposition algorithm of TOPSIS for solving (TL-LSLMOP-SP)$_{\text{rhs}}$ of block angular structure

Thus, we can introduce the following decomposition algorithm of TOPSIS method to gernerate a set of satisfactory solutions for (TL-LSLMOP-SP)$_{\text{rhs}}$ of block angular structure:

The algorithm (Alg-I):

Phase (0):

Step 1. Transform problem (1) to the form of problem (3).

Step 2. Transform problem (3) to the form of problem (8).

Phase (I):

Step 3. Construct the PIS payoff table of problem (9) by using the decomposition algorithm [27, 29, 43], and obtain $z^{*\text{FLDM}} = \left(z_1^{*FLDM}, z_2^{*FLDM}, \ldots, z_{k_{I_1}}^{*FLDM}\right)$ the individual positive ideal solutions.

Step 4. Construct the NIS payoff table of problem (9) by using the decomposition algorithm, and obtain $z^{-\text{FLDM}} = (z_1^{-FLDM}, z_2^{-FLDM}, \ldots\ldots, z_{k_{I_1}}^{-FLDM})$, the individual negative ideal solutions.

Step 5. Use equations (10 & 11) and the above steps (3 & 4) to construct $d_p^{PIS\,\text{FLDM}}$ and $d_p^{NIS\,\text{FLDM}}$.

Step 6. Ask the FLDM to select
p= p* ∈ {1, 2, …, ∞ },

Step 7. Ask the FLDM to select $w_i = w_i^*, i = 1,2,\dots,k_{I_1}$, where $\sum_{i=1}^{k_{I_1}} w_i = 1$,

Step 8. Use steps (4 & 6) to compute $d_p^{PIS^{\text{FLDM}}}$ and $d_p^{NIS^{\text{FLDM}}}$.

Step 9. Transform problem (9) to the form of problem (12).

Step 10. Construct the payoff table of problem (12):
At *p = 1*, use the decomposition algorithm [27, 29, 43].
At *p ≥ 2*, use the generalized reduced gradient method, [43, 44], and obtain:
$d_p^{-FLDM} = \left(\left(d_p^{PIS^{\text{FLDM}}}\right)^-, \left(d_p^{NIS^{\text{FLDM}}}\right)^-\right)$, $d_p^{*FLDM} = \left(\left(d_p^{PIS^{\text{FLDM}}}\right)^*, \left(d_p^{NIS^{\text{FLDM}}}\right)^*\right)$.

Step 11. Construct problem (15) by using the membership functions (13).

Step 12. Solve problem (15) to obtain $\left(\gamma^{*FLDM}, X^{*\text{FLDM}}\right)$.

Step 13. Ask the FLDM to select the maximum negative and positive tolerance values τ_i^L and τ_i^R, $i = 1,2,\dots,n_{I_1}$ on the decision vector $X_{I_1}^{*FLDM} = \left(x_{I_1 1}^{*FLDM}, x_{I_1 2}^{*FLDM}, \dots, x_{I_1 n_{I_1}}^{*FLDM}\right)$.

Phase (II):

Step 14. Construct the PIS payoff table of problem (17) by using the decomposition algorithm [27, 29, 43], and obtain $z^{*\text{SLDM}} = \left(z_1^{*SLDM}, z_2^{*SLDM}, \dots, z_{k_{I_2}}^{*SLDM}\right)$ the individual positive ideal solutions.

Step 15. Construct the NIS payoff table of problem (17) by using the decomposition algorithm, and obtain $z^{-\text{SLDM}} = (z_1^{-\text{SLDM}}, z_2^{-\text{SLDM}}, \dots, z_{k_{I_2}}^{-\text{SLDM}})$ the individual negative ideal solutions.

Step 16. Use equations (18 & 19) and the above steps (14 & 15) to construct $d_p^{PIS^{\text{TL}}}$ and $d_p^{NIS^{\text{TL}}}$.

Step 17. Ask the FLDM to select $w_i = w_i^*, i = 1,2,\dots,k$, where $\sum_{i=1}^{k} w_i = 1$,

Step 18. Use steps (14 , 15 & 16) to compute $d_p^{PIS^{\text{TL}}}$ and $d_p^{NIS^{\text{TL}}}$.

Step 19. Transform problem (8) to the form of problem (20).

Step 20. Construct the payoff table of problem (20):
At p=1,use the decomposition algorithm [27, 29, 43],
At p≥2, use the generalized reduced gradient method, [46, 47], and obtain:
$d_p^{-\text{TL}} = \left(\left(d_p^{PIS^{\text{TL}}}\right)^-, \left(d_p^{NIS^{\text{TL}}}\right)^-\right)$, $d_p^{*TL} = \left(\left(d_p^{PIS^{\text{TL}}}\right)^*, \left(d_p^{NIS^{\text{TL}}}\right)^*\right)$.

Step 21. Use equations (13 and 18) to construct problem (23).

Step 22. Solve problem (23) to obtain $\left(\gamma^{*\text{TL}}, X^{*\text{TL}}\right)$.

Step 23. If the FLDM is satisfied with the current solution , go to step 24. Otherwise, go to step 6.

Step 24. Stop.

VI. An illustrative numerical example

Consider the following (TL-LSLMOP-SP)$_{\text{rhs}}$ of block angular structure:

[FLDM]

$$\underset{x_1, x_2}{Maximize}\ f_{11}(X) = 4x_1 + 6x_2 - x_3 + 7x_4 \quad (24-1)$$

$$\underset{x_1, x_2}{Maximize}\ f_{12}(X) = -2x_1 + 9x_2 + 13x_3 + x_4 \quad (24-2)$$

$$\underset{x_1, x_2}{Minimize}\ f_{13}(X) = -x_1 + 3x_2 - x_3 + x_4 \quad (24-3)$$

$$\underset{x_1, x_2}{Minimize}\ f_{14}(X) = 6x_1 - 2x_2 + x_3 + x_4 \quad (24-4)$$

$$where\ x_1, x_2\ solves\ the\ second\ level$$

[SLDM]

$$\underset{x_3, x_4}{Maximize}\ f_{21}(X) = x_1 - 5x_2 + 3x_3 + 19x_4 \quad (24-5)$$

$$\underset{x_3, x_4}{Minimize}\ f_{22}(X) = 4x_1 + x_2 + x_3 - x_4 \quad (24-6)$$

subject to

$$P\{x_1 + x_2 + x_3 + x_4 \le 6v_1\} \ge 0.7257, \quad (24-7)$$

$$P\{5x_1 + x_2 \le 7v_2\} \ge 0.5, \quad (24-8)$$

$$P\{x_3 + x_4 \leq 8v_3\} \geq 0.9015, \quad (24-9)$$

$$x_1, x_2, x_3, x_4 \geq 0 \quad (24-10)$$

Suppose that $v_i, i = 1,2,3$ are linearly independent normal distributed parameters with the following means and variances: $E(v_1) = 8,\ E(v_2) = 2, E(v_3) = 1\ , Var(v_1) = 25, Var(v_2) = 4, Var(v_3) = 25$.
Solution:
By using problem (3), we can have
[FLDM]

$$\underset{x_1\ ,x_2}{Maximize}\ f_{11}(X) = 4x_1 + 6x_2 - x_3 + 7x_4 \quad (25-1)$$

$$\underset{x_1\ ,x_2}{Maximize}\ f_{12}(X) = -2x_1 + 9x_2 + 13x_3 + x_4 \quad (25-2)$$

$$\underset{x_1\ ,x_2}{Minimize}\ f_{13}(X) = -x_1 + 3x_2 - x_3 + x_4 \quad (25-3)$$

$$\underset{x_1\ ,x_2}{Minimize}\ f_{14}(X) = 6x_1 - 2x_2 + x_3 + x_4 \quad (25-4)$$

$$where\ x_1, x_2\ solves\ the\ second\ level$$

[SLDM]

$$\underset{x_3\ ,x_4}{Maximize}\ f_{21}(X) = x_1 - 5x_2 + 3x_3 + 19x_4 \quad (25-5)$$

$$\underset{x_3\ ,x_4}{Minimize}\ f_{22}(X) = 4x_1 + x_2 + x_3 - x_4 \quad (25-6)$$

subject to

$$x_1 + x_2 + x_3 + x_4 \leq 66, \quad (25-7)$$

$$5x_1 + x_2 \leq 14, \quad (25-8)$$

$$x_3 + x_4 \leq 59.6, \quad (25-9)$$

$$x_1, x_2, x_3, x_4 \geq 0 \quad (25-10)$$

- Obtain PIS and NIS payoff tables for the FLDM of problem (25).

Table (1) : *PIS* payoff table for the FLDM of problem (25)

	f_{11}	f_{12}	f_{13}	f_{14}	x_1	x_2	x_3	x_4
$\underset{x_1\ ,x_2}{Maximize}\ f_{11}(X)$	*455.6*	*117.2*	*78.8*	*46.8*	*0*	*6.4*	*0*	*59.6*
$\underset{x_1\ ,x_2}{Maximize}\ f_{12}(X)$	-21.2	832.4	*-40.8*	*46.8*	*0*	*6.4*	59.6	0
$\underset{x_1\ ,x_2}{Minimize}\ f_{13}(X)$	*-48.4*	*769.2*	*-62.4*	*76.4*	*2.8*	*0*	*59.6*	*0*
$\underset{x_1\ ,x_2}{Minimize}\ f_{14}(X)$	*84*	*126*	*42*	*-14*	*0*	*14*	*0*	*0*

PIS: $f^{*\text{FLDM}}$ = (455.6 , 832.4 , -62.4 , -14)

Table (2) : *NIS* payoff table for the FLDM of problem (25)

	f_{11}	f_{12}	f_{13}	f_{14}	x_1	x_2	x_3	x_4
$\underset{x_1\ ,x_2}{Minimize}\ f_{11}(X)$	*-59.6*⁻	*774.8*	*-59.6*	*-59.6*	*0*	*0*	*59.6*	*0*
$\underset{x_1\ ,x_2}{Minimize}\ f_{12}(X)$	11.2	-5.6 ⁻	*-2.8*	*16.8*	*2.8*	*0*	0	0
$\underset{x_1\ ,x_2}{Maximize}\ f_{13}(X)$	*448*	*178*	*94* ⁻	*24*	*0*	*14*	*0*	*52*
$\underset{x_1\ ,x_2}{Maximize}\ f_{14}(X)$	*428.4*	*54*	*56.8*	*136* ⁻	*2.8*	*0*	*0*	*59.6*

NIS: $f^{-\text{FLDM}}$ = (*-59.6* , -5.6 , *94* , *136*)

- Next, compute equation (11) and obtain the following equations:

$$d_P^{PIS^{\text{FLDM}}} = \left[w_{11}^p \left(\frac{455.6 - f_{11}(X)}{455.6 - (-59.6)} \right)^p + w_{12}^p \left(\frac{832.4 - f_{12}(X)}{832.4 - (-5.6)} \right)^p + w_{13}^p \left(\frac{f_{13}(x) - (-62.4)}{94 - (-62.4)} \right)^p + w_{14}^p \left(\frac{f_{14}(x) - (-14)}{136 - (-14)} \right)^p \right]^{1/p}$$

$$d_P^{NIS^{\text{FLDM}}} = \left[w_{11}^p \left(\frac{f_{11}(X) - (-59.6)}{455.6 - (-59.6)} \right)^p + w_{12}^p \left(\frac{f_{12}(X) - (-5.6)}{832.4 - (-5.6)} \right)^p + w_{13}^p \left(\frac{94 - f_{13}(x)}{94 - (-62.4)} \right)^p + w_{14}^p \left(\frac{136 - f_{14}(x)}{136 - (-14)} \right)^p \right]^{1/p}$$

- Thus, problem (12) is obtained.
- In order to get numerical solutions, assume that $w_{11}^p = w_{12}^p = w_{13}^p = w_{14}^p = 0.25$ and *p=2,*

Table (3) : *PIS* payoff table of problem (12), when *p=2.*

	$d_2^{PIS^{\text{FLDM}}}$	$d_2^{NIS^{\text{FLDM}}}$	x_1	x_2	x_3	x_4
$Min.\, d_2^{PIS^{\text{FLDM}}}$	*0.2182**	*0.2945*-	*0*	*10.8954*	*35.2202*	*19.8844*
$Max.\, d_2^{NIS^{\text{FLDM}}}$	*0.3029*-	*0.2269**	*2.8*	*0*	*6.0063*	*27.1433*

d_2^{*FLDM} = *(0.2182 , 0.2269)* , d_2^{-FLDM} = *(0.3029 , 0.2945).*

- Now, it is easy to compute (15) :

$Maximize\ \gamma^{\text{FLDM}}$
subject to

$$x_1 + x_2 + x_3 + x_4 \le 66,$$
$$5x_1 + x_2 \le 14,$$
$$x_3 + x_4 \le 59.6,$$
$$\left(\frac{d_2^{PIS^{\text{FLDM}}}(X) - 0.2182}{0.3029 - 0.2182} \right) \ge \gamma^{\text{FLDM}},$$
$$\left(\frac{0.2269 - d_2^{NIS^{\text{FLDM}}}(X)}{0.2269 - 0.2945} \right) \ge \gamma^{\text{FLDM}},$$
$$x_1, x_2, x_3, x_4 \ge 0, \qquad \gamma^{\text{FLDM}} \in [0,1] .$$

The maximum “satisfactory level” (γ^{FLDM} *=1*) is achieved for the solution X_1^{*FLDM} *=0* , X_2^{*FLDM} *=13.2092* , X_3^{*FLDM} *=0,* X_4^{*FLDM} *=0* and $(f_{11}, f_{12}, f_{13}, f_{14}) = (79.2552\ , 118.8828\ , 39.6276\ , -26.4184)$. Let the FLDM decide X_1^{*FLDM} *= 0* and X_2^{*FLDM} *= 13.2092* with positive tolerance $\tau^R = 0.5\ and\ \tau^L = 0.5$.

- Obtain *PIS* and *NIS* payoff tables for the SLDM of Problem (25).

Table (4) : *PIS* payoff table for the SLDM of problem (25)

	f_{21}	f_{22}	x_1	x_2	x_3	x_4
$\underset{x_3\ ,x_4}{Maximize}\ f_{21}(X)$	*1135.2**	*-48.4*	*2.8*	*0*	*0*	*59.6*
$\underset{x_3\ ,x_4}{Minimize}\ f_{22}(X)$	*1132.4*	*-59.6**	*0*	*0*	*0*	*59.6*

PIS: $f^{*\text{SLDM}}$ =(*1135.2* , *-59.6*)

Table (5) : *NIS* payoff table for the SLDM of problem (24)

	f_{21}	f_{22}	x_1	x_2	x_3	x_4
$\underset{x_3\ ,x_4}{Minimize}\ f_{21}(X)$	*-70* -	*14*	*0*	*14*	*0*	*0*
$\underset{x_3\ ,x_4}{Maximize}\ f_{22}(X)$	*158.2*	*71.7* -	*1.9*	*4.5*	*59.6*	*0*

NIS: $f^{-\text{SLDM}}$ =(*-70* , *71.7*)

- Next, compute equation (19) and obtain the following equations:

$$d_P^{PIS^{\text{TL}}} = \left[w_{11}^p \left(\frac{455.6 - f_{11}(X)}{455.6 - (-59.6)}\right)^p + w_{12}^p \left(\frac{832.4 - f_{12}(X)}{832.4 - (-5.6)}\right)^p + w_{13}^p \left(\frac{f_{13}(x) - (-62.4)}{94 - (-62.4)}\right)^p + w_{14}^p \left(\frac{f_{14}(x) - (-14)}{136 - (-14)}\right)^p + w_{25}^p \left(\frac{1135.2 - f_{21}(X)}{1135.2 - (-70)}\right)^p + w_{26}^p \left(\frac{f_{22}(X) - (-59.6)}{71.7 - (-59.6)}\right)^p\right]^{1/p}$$

$$d_P^{NIS^{\text{TL}}} = \left[w_{11}^p \left(\frac{f_{11}(X) - (-59.6)}{455.6 - (-59.6)}\right)^p + w_{12}^p \left(\frac{f_{12}(X) - (-5.6)}{832.4 - (-5.6)}\right)^p + w_{13}^p \left(\frac{94 - f_{13}(x)}{94 - (-62.4)}\right)^p + w_{14}^p \left(\frac{136 - f_{14}(x)}{136 - (-14)}\right)^p + w_{25}^p \left(\frac{f_{21}(X) - (-70)}{1135.2 - (-70)}\right)^p + w_{26}^p \left(\frac{71.7 - f_{22}(X)}{71.7 - (-59.6)}\right)^p\right]^{1/p}$$

- Thus, problem (20) is obtained.
- In order to get numerical solutions, assume that $w_{21}^p=0.2$, $w_{22}^p=0.2$, $w_{23}^p=0.2$, $w_{24}^p=0.2$, $w_{25}^p = 0.1$, $w_{26}^p=0.1$ and $p=2$,

Table (6) : *PIS* payoff table of problem (20), when *p=2*.

	$d_2^{PIS^{\text{BL}}}$	$d_2^{NIS^{\text{BL}}}$	x_1	x_2	x_3	x_4
$Min.\, d_2^{PIS^{\text{BL}}}$	0.1934^*	0.2329^-	0	6.4	31.8117	27.7883
$Max.\, d_2^{NIS^{\text{BL}}}$	0.2936^-	0.2257^*	0	1.6426	0.4845	0

d_2^{*BL} =(0.1934 , 0.2257) , d_2^{-BL} =(0.2936 , 0.2329).

- Now, it is easy to compute (23) :

$Maximize\ \gamma^{\text{BL}}$
subject to

$$x_1 + x_2 + x_3 + x_4 \le 66,$$
$$5x_1 + x_2 \le 14,$$
$$x_3 + x_4 \le 59.6,$$
$$\left(\frac{d_2^{PIS^{\text{BL}}}(X) - 0.1934}{0.2936 - 0.1934}\right) \ge \gamma^{\text{BL}},$$
$$\left(\frac{0.2257 - d_2^{NIS^{\text{BL}}}(X)}{0.2257 - 0.2329}\right) \ge \gamma^{\text{BL}},$$
$$\left(\frac{(0 + 0.5) - x_1}{0.5}\right) \ge \gamma^{\text{BL}},$$
$$\left(\frac{x_1 - (0 - 0.5)}{0.5}\right) \ge \gamma^{\text{BL}},$$
$$\left(\frac{(13.2092 + 0.5) - x_2}{0.5}\right) \ge \gamma^{\text{BL}},$$
$$\left(\frac{x_2 - (13.2092 - 0.5)}{0.5}\right) \ge \gamma^{\text{BL}},$$
$$x_1, x_2, x_3, x_4 \ge 0, \quad \gamma^{\text{BL}} \in [0,1] .$$

The maximum “satisfactory level” (δ^{BL} = *0.9042*) is achieved for the solution $X_1^{*BL} = 0$, $X_2^{*BL} = 13.1613$, $X_3^{*BL} = 0$, $X_4^{*BL} = 0$.

VI. Summary and Concluding remarks

In this paper, a TOPSIS approach has been extended to solve $(\text{TL-LSLMOP-SP})_{\text{rhs}}$ of block angular structure. The $(\text{TL-LSLMOP-SP})_{\text{rhs}}$ of block angular structure using TOPSIS approach provides an effective way to find the compromise (satisfactory) solution of such problems. In order to obtain a compromise (

satisfactory) integer solution to the $(TL\text{-}LSLMOP\text{-}SP)_{rhs}$ of block angular structure using the proposed TOPSIS approach, a modified formulas for the distance function from the PIS and the distance function from the NIS are proposed and modeled to include all objective functions of both the first and the second levels. Thus, the two-objective problem is obtained which can be solved by using membership functions of fuzzy set theory to represent the satisfaction level for both criteria and obtain TOPSIS, compromise solution by a second–order compromise. The max-min operator is then considered as a suitable one to resolve the conflict between the new criteria (the shortest distance from the *PIS* and the longest distance from the NIS). An interactive TOPSIS algorithm for solving these problems are also proposed. It is based on the decomposition algorithm of $(TL\text{-}LSLMOP\text{-}SP)_{rhs}$ of block angular structure via TOPSIS approach, [5]. This algorithm has few features, (i) it combines both $(TL\text{-}LSLMOP\text{-}SP)_{rhs}$ of block angular structure and TOPSIS approach to obtain TOPSIS's compromise solution of the problem, (ii) it can be efficiently coded. (iii) it was found that the decomposition based method generally met with better results than the traditional simplex-based methods. Especially, the efficiency of the decomposition-based method increased sharply with the scale of the problem. An illustrative numerical example is given to demonstrate the proposed TOPSIS approach and the decomposition algorithm.

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